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The Queensland College of Teachers has commissioned the Australian Council for Educational Research to prepare a series of electronic Research Digests.

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The Research Digest is available in HTML and PDF versions on the Queensland College of Teachers' website at www.qct.edu.au

The Research Digests

This Research Digest is one of a series of periodic digests produced by the Australian Council for Educational Research (ACER) for the Queensland College of Teachers. Each digest focuses on a single topical issue, and provides a review of major messages from research on the issue. A key feature of the Digests is an emphasis on what the research means for teachers and teaching. Over the course of several editions, a wide range of issues is covered, so that teachers from different areas of schooling can find topics of relevance to their needs and interests.

Previous Issues

1. Writing to learn, October 2007
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9. Teaching Critical Thinking, April 2013
10. Parents and teachers: Working together to foster children's learning, November 2014

Big Ideas in Mathematics Teaching

The idea of Big Ideas in mathematics education has now been around for a while, but what does it mean for the teaching and learning of mathematics in schools? This Digest attempts to address this issue by describing two views about Big Ideas, and aims to give some insights into how the Big Ideas concept can be used by mathematics teachers.

AITSL Standards

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Big Ideas in Mathematics Teaching explicitly addresses a number of the Australian Professional Standards for Teachers. (<http://www.aitsl.edu.au/australian-professional-standards-for-teachers/standards/overview/organisation-of-the-standards>)

The domains of teaching and the related Standards and focus areas covered are shown below.

Domain of teaching	Standard	Focus area
Professional Knowledge	1. Know students and how they learn	1.2 Understand how students learn 1.5 Differentiate teaching to meet the specific learning needs of students across the full range of abilities
	2. Know the content and how to teach it	2.1 Content and teaching strategies of the teaching area 2.2 Content selection and organisation 2.3 Curriculum, assessment and reporting 2.5 Literacy and numeracy strategies
Professional Practice	3. Plan for and implement effective teaching and learning	3.1 Establish challenging learning goals 3.2 Plan, structure and sequence learning programs 3.3 Use teaching strategies

What are Big Ideas and why are they important in teaching mathematics?

Initially, the term 'Big Ideas' referred to how mathematical information can be classified in different ways compared with the traditional school mathematics curriculum content areas. Steen (1990), for example, identified six broad categories pertaining to: Quantity, Dimension, Pattern, Shape, Uncertainty, and Change. Rutherford & Ahlgren (1989) described networks of related ideas: Numbers, Shapes, Uncertainty, Summarizing data, Sampling, and Reasoning. These perspectives provide an overarching and integrative idea of mathematics and how mathematics is used in the world—that is, they are about framing and viewing mathematics as making connections with the real world.

The national standards of the National Council of Teachers of Mathematics in the US also referred to big ideas:

Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise. (National Council of Teachers of Mathematics, 2000, p. 17)

Big Ideas were seen as a potential vehicle for making mathematics education a more integrated study. Descriptions of effective teaching of mathematics (Ma, 1999; Sullivan, 2011) and related research (Askew, Brown, Rhodes, William & Johnson, 1997; Clarke & Clarke, 2004; Boaler & Humphreys, 2005) also consistently refer to the need for teachers to have a sense of how mathematics is a coherent and connected whole. Ma and Kessel described the need for teachers to have a profound understanding of fundamental mathematics which was:

... not in terms of knowledge of facts, skills and procedures, but rather in terms of four key features: connectedness, multiple perspectives, recurring basic ideas, and longitudinal coherence. (Ma & Kessel, 2003)

A similar view is expressed in the rationale for the *Australian Curriculum: Mathematics* (Australian Curriculum Assessment and Reporting Authority, (2013):

Mathematics is composed of multiple but interrelated and interdependent concepts and systems which students apply beyond the mathematics classroom. (<http://www.australiancurriculum.edu.au/mathematics/rationale>)

By its emphasis on the four proficiency strands of Understanding, Fluency, Problem Solving and Reasoning, the *Australian Curriculum: Mathematics* seeks to shift our thinking from a narrow focus on content to potentially a Big Ideas view of learning in mathematics. An example of how this change can be implemented in classroom practice and reporting by the Queensland Curriculum and Assessment Authority (QCAA) is discussed further in Scenario 3 below.

In the *Effective Teachers of Numeracy* study (Askew et al., 1997), this view was also supported, with highly effective teachers believing that being numerate requires

What are Big Ideas and why are they important in teaching mathematics?

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'having a rich network of connections between different mathematical ideas.' This is in contrast to ways in which mathematical content knowledge for teachers is reduced to lists of specific dot points in curriculum frameworks that Askew terms 'death by a thousand bullet points', saying that 'too much effort goes into specifying the knowledge that teachers need to know' (Askew, 2008, p. 21).

Crucial questions from a teaching perspective are about how BIG Big Ideas are and how many of them can there be? If a Big Idea is too big, how useful will it be? It is important that Big Ideas are not described and articulated along traditional curriculum content strands—Big Ideas are BIG because they run across strands. Any list of Big Ideas should not be too long—the idea is that content knowledge and teaching practices should be based on a small number of overarching ideas. This is what helps make the concept of Big Ideas potentially powerful. As described above, for effective teaching the concept of Big Ideas allows the mathematics to be coherent for the teacher but more importantly it enables students to develop a deep understanding of mathematics and its applications.

Again, the *Australian Curriculum: Mathematics* reinforces this view. For example in the Overview of Years 7–10:

... students of this age also need an understanding of the connections between mathematical concepts and their application in their world as a motivation to learn. This means using contexts directly related to topics of relevance and interest to this age group.

Clearly there are seen to be many advantages to knowing about the Big Ideas in mathematics and using them in one's teaching. However, recent research in Germany, the *Awareness of Big Ideas in Mathematics Classrooms* project (Kuntze et al., 2011) and Australia, *Important Ideas in Mathematics: What are they and where do you get them?* (Clarke, Clarke & Sullivan, 2012), has revealed that teachers, pre-service and in-service, need support in identifying, describing or linking important ideas in mathematics.

This is particularly the case when many secondary mathematics classes are being taken by out-of-field teachers, and when many generalist primary teachers have not yet had the opportunity to deepen their conceptual understanding.

In an attempt to address this issue, in the following sections we elaborate on two views about Big Ideas—the overarching and integrative view of describing mathematics (a top-down view), and the bottom-up analysis which looks at a more clustered curriculum-based approach. We hope this will give some insights into how the Big Ideas concept can be used by mathematics teachers.

►► ACTION

Download a copy of 'On the shoulders of giants: New approaches to numeracy' and use each essay as the basis of a school department discussion about Big Ideas. How could you apply the ideas in your teaching?

<http://www.stolaf.edu/people/steen/Papers/90shoulders.pdf>

... students of this age also need an understanding of the connections between mathematical concepts and their application in their world as a motivation to learn. This means using contexts directly related to topics of relevance and interest to this age group.

(Australian Curriculum)

Overarching and integrative Big Ideas: a top-down view from a real-world perspective

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Overarching and integrative Big Ideas: a top-down view from a real-world perspective

In one of the influential collections of papers about Big Ideas, *On the shoulders of giants: New approaches to numeracy* (Steen, 1990), the writers argue that mathematics must be grounded in our intuition and experience of the world:

In this volume readers will find a vision of the richness of mathematics ... that illustrates different possible strands of school mathematics. These papers expand on the theme of mathematics as the language and science of patterns and are introduced by a brief essay that highlights interconnections and common ideas. ... They ... suggest through numerous imaginative examples how mathematical ideas can be developed from informal childhood exploration through formal school and college study. The papers in this volume are intended as a vehicle to stimulate creative approaches to mathematics curricula in the next century. (Steen, 1990, pp.iii-iv)

In this collection, six broad categories were developed and described by six different authors: Quantity, Dimension, Pattern, Shape, Uncertainty, and Change.

Big Idea and author	Brief description
Pattern: Lynn Arthur Steen	<i>Pattern</i> is seen as a wide-ranging concept that covers patterns encountered all around us, such as those in musical forms, nature, traffic patterns, etc. It is argued that our ability to recognize, interpret, and create patterns is the key to dealing with the world around us.
Dimension: Thomas F. Banchoff	<i>Dimension</i> includes “big ideas” related to one, two, and three dimensions of “things” (using spatial and numerical descriptions), projections, lengths, perimeters, planes, surfaces, location, etc. Facility with each dimension requires a sense of “benchmarks” and estimation, direct measurement and derived measurement skills.
Quantity: James T. Fey	<i>Quantity</i> is described as an outgrowth of people’s need to quantify the world around us, using attributes such as: length, area, and volume of rivers or land masses; temperature, humidity, and pressure of our atmosphere; populations and growth rates of species; motions of tides; revenues or profits of companies, etc.
Uncertainty: David S. Moore	<i>Uncertainty</i> covers “big ideas” related to probability, subjective probability, and relevant statistical methods. Few things in the world are 100% certain; therefore the ability to attach a number that represents the likelihood of an instance is a valuable tool whether it has to do with the weather, the stock-market, or the decision to board a plane. It also covers “big ideas” such as variability, sampling, error, or prediction, and related statistical topics such as data collection, data displays, and graphs.
Shape: Majorie Senechal	<i>Shape</i> is a category describing real images and entities that can be visualized (e.g., houses and buildings, designs in art and craft, safety signs, packaging, snowflakes, knots, crystals, shadows and plants), as well as highly abstract “things” greater than three dimensions.
Change: Ian Stewart	<i>Change</i> describes the mathematics of how the individual organisms grow, populations vary, prices fluctuate, objects travelling speed up and slow down. Change and rates of change help provide a narration of the world as time marches on. Additive, multiplicative, exponential patterns of change can characterize steady trends; periodic changes suggest cycles and irregular change patterns connect with chaos theory.

Most importantly, they [pre-service teachers] noted how the ‘big ideas’ focus enabled them to help children better through using links and connections to different concepts to overcome misconceptions and misunderstandings.

(Hurst)

Scenario 1

A real-world problem-solving framework

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Scenario 1. A real-world problem-solving framework

One illustration of a framework that has attempted to use and implement this approach is the OECD's Program for International Student Assessment (PISA). The original conception of mathematical literacy developed for PISA acknowledged the need to develop and hence assess students' capacity to transfer and apply their knowledge and skills to problems that originate outside school-based learning contexts. A core idea behind mathematical literacy in PISA is mathematical modelling, which assumes that when individuals use mathematics and mathematical tools to solve problems set in a real-world context, they work their way through a series of stages depicted in Figure 1 (OECD, 2013, p26).

The processes outlined of formulating, employing, interpreting, and evaluating mathematics are key components of the mathematical modelling cycle that has underpinned the mathematical literacy construct in PISA since its beginnings and has been elaborated and enhanced in the description and definition for PISA 2012. These processes each draw on the problem solver's fundamental mathematical capabilities (another set of Big Ideas found in the PISA framework) and on his or her mathematical knowledge in four overarching Big Idea content areas: Quantity, Uncertainty & data, Change & relationships, and Space & shape.

PISA has attempted to operationalise the connections between the world of mathematics and the real world in a school-based assessment program—one model of the overarching and integrative big ideas model.

In relation to Australia's performance in PISA, no matter how you read the results in relation to the mathematical skills of our young people, the reality is that too many children who study mathematics in school are unable to easily transfer formal mathematical skills and apply them to tasks that are embedded in or resemble real-world contexts. The PISA mathematical literacy construct and its items provide a vehicle for addressing this issue in the classroom. In PISA the aim is to find real situations and adapt them for the assessment purpose. PISA test developers find real materials or contexts and simplify them so they are accessible and still realistic. Teachers can model this in their own classrooms.

►► ACTION

Download the publicly released PISA items from: www.oecd.org/pisa/pisaproducts/pisa2012-2006-rel-items-maths-ENG.pdf

Use these as models for developing similar classroom questions you can use with your learners – try to find real world instances of maths and adapt them.



Challenge in real-world context

Mathematical content categories: Quantity; Uncertainty & data; Change & relationships; Space & shape

Real-world context categories: Personal; Societal; Occupational; Scientific

Mathematical thought and action

Mathematical concepts, knowledge and skills

Fundamental mathematical capabilities: Communication; Representation; Devising strategies; Mathematisation; Reasoning and argument; Using symbolic, formal and technical language and operations; Using mathematical tools

Processes: Formulate, Employ, Interpret/Evaluate

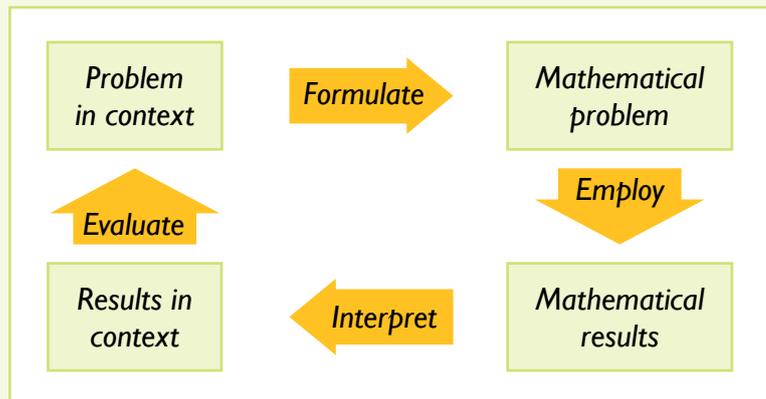


Figure 1. A model of mathematical literacy in practice for PISA 2012

Scenario 2

How it can work in teaching and learning

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Scenario 2. How it can work in teaching and learning

An existing classroom approach that matches the PISA mathematical modelling cycle of formulating, employing, interpreting, and evaluating mathematics is that of Newman (1977). She developed a structured method of Error Analysis related to the stages of solving a word problem. This approach is highlighted by the NSW Department of Education and Communities: <http://www.curriculumsupport.education.nsw.gov.au/secondary/mathematics/numeracy/newman/index.htm>

Newman's five stages to problem solving align with the PISA cycle:

- | | |
|--|---|
| 1. READING the problem | |
| 2. COMPREHENSION of the problem in your own words | |
| 3. TRANSFORMATION of the problem from the real world to the mathematics world by identifying the mathematics that will be used | ▶ The first 3 stages align with the PISA FORMULATE process |
| 4. PROCESS SKILLS of actually carrying out the mathematics correctly | ▶ The 4th stage aligns with the PISA EMPLOY process |
| 5. ENCODING the result from the mathematics world back to making sense of it in the real world. | ▶ The 5th stage aligns with the PISA INTERPRET/EVALUATE process |

The Newman model is of an interview between teacher and student in which the teacher's questions prompt the student to link the parts of a solution together.

Example problem:

Jim is packing cans into boxes. He has 40 cans to pack. Each box holds 6 cans. How many boxes does Jim need to put all the cans into boxes?



Stage	Teacher prompt	Student response
Reading	"Please read the question to me. If you don't know a word, leave it out".	"Jim is packing cans into boxes. He has 40 cans to pack. Each box holds 6 cans. How many boxes does Jim need to put all the cans into boxes?"
Comprehension	"Tell me what the question is asking you to do".	"So we don't know how many boxes, but the cans need to go into the boxes in groups of 6".
Transformation	"Tell me how you are going to find the answer".	"To split 40 into 6 groups I'll have to divide 40 by 6".
Process skills	"Show me what to do to get the answer. Talk aloud as you do it, so that I can understand how you are thinking".	"40 divided by 6 is the same as 20 divided by 3, so it's a bit less than 7".
Encoding	"Now, write down your answer to the question".	"6 boxes won't take all the cans, so it needs to be 7 boxes".



Newman discovered from close study and interviews with school students that more than 50% of the barriers to numeracy performance were related to the first three of the five stages, before any formal mathematics process is employed. This result has been replicated in other countries and settings (White, 2009).

... Newman's research consistently pointed to the inappropriateness of many remedial mathematics programs in schools in which the revision of standard algorithms was overemphasised, while hardly any attention was given to difficulties associated with Comprehension and Transformation (Ellerton & Clements, 1996, p. 186)

►► ACTION

Use the Newman method in your classroom based on the PISA publicly released items referred to above (if appropriate for your level). Start by using the five stages with the whole class first to model the process. Then get learners to use the method together in pairs or threes. You can give groups of learners differentiated problems that are targeted at their various levels of skill, ability and interest.

From Atomised to Clustered curriculum: a bottom-up analysis

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From Atomised to Clustered curriculum: a bottom-up analysis

An alternative approach to using Big Ideas in mathematics education is to accept the atomised list of dot points such as frequently exists in curriculum frameworks and work from the bottom up, by organising or grouping them according to some structure or system based on professional judgement.

A good example of this is offered by Charles (2005) in a most useful paper, 'Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics', where he argues that Big Ideas should be the foundation for both the content knowledge and daily practice of teachers, for the way they provide coherence and focus to the day-to-day practice of the classroom craft. This is because Big Ideas promote connections, connections lead to better understanding, and understanding promotes memory and enhances transfer. He describes a process of sifting and sorting the curriculum content that he was given to teach by looking across content strands rather than by grade level, and identifying 'clusters of math understandings, ideas that seemed to be connected to something bigger'. He suggests 21 big ideas:

- ▶ Numbers
- ▶ Base 10 Numeration
- ▶ Equivalence
- ▶ Comparison
- ▶ Operations & relationships
- ▶ Properties
- ▶ Basic facts & Algorithms
- ▶ Estimation
- ▶ Patterns
- ▶ Variables
- ▶ Proportionality
- ▶ Relations & Functions
- ▶ Equations & Inequalities
- ▶ Shapes and Solids
- ▶ Orientation and Location
- ▶ Transformations
- ▶ Measurement
- ▶ Data collection
- ▶ Data representation
- ▶ Data Distribution
- ▶ Chance

For example, his Big Idea #3 is **Equivalence**: *Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.* He elaborates using examples from Whole Number (2 hundreds 4 tens is equivalent to 24 tens), Fractions (an infinite set of equivalent fractions), Algebra (expanded and factorised forms of an expression), and Measurement (in equivalent ways using different units). This Big Idea connects the longitudinal development of both curriculum content and student learning, so that teachers and students are not starting from scratch each year in the learning progression.

Each teacher might look at Charles's list and see other clusters that could be further grouped or some items that they would split. So be it, as the aim of the exercise is to give the teacher a secure mental frame of reference,

▶▶ ACTION

Access a copy of Charles's paper, 'Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics' and as a school staff or individual review his 21 Big Ideas.

Does this fit the Australian Curriculum?

How else could you have grouped or distilled the curriculum into a set of Big Ideas?

Do you agree with Mulligan and Mitchelmore that Pattern and Structure is a key Big Idea?



or *mathematical sensibility* (Askew, 2008), within which to plan and conduct their learning sequences.

Some recent research with pre-service teachers in Western Australia (Hurst, 2014) reports on the positive effects of challenging the candidates to make a Big Ideas review via concept maps of their understanding of curriculum linkages.

... as the unit progressed, they began to feel genuinely excited by the prospect of thinking about mathematics and teaching it in this way. Most importantly, they noted how the 'big ideas' focus enabled them to help children better through using links and connections to different concepts to overcome misconceptions and misunderstandings. (p.293).

Mulligan and Mitchelmore (2009) argue that **Pattern and Structure** has the capacity, particularly in the early years, to be the one Big Idea that subsumes all the others in Charles's list. They

... propose a new construct, Awareness of Mathematical Pattern and Structure (AMPS), which generalises across early mathematical concepts, can be reliably measured, and is correlated with mathematical understanding. (p. 34)

Scenario 3

Using the Proficiencies in the Australian Curriculum: Mathematics

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Scenario 3. Using the Proficiencies in the Australian Curriculum: Mathematics

An example of how the Proficiencies have been used as the basis for making connections between the atomised content and elaborations of the *Australian Curriculum: Mathematics* has been developed by the Queensland Curriculum and Assessment Authority (QCAA). They have put the focus on Proficiencies by making them the basis of teacher assessment records and reports to parents. This builds the expectation that the activities planned for a learning sequence provide opportunities for students to demonstrate and teachers to observe the Proficiencies in action. For the purpose of developing reports it provides a rubric to connect Understanding & Fluency (seen as a pair) and Problem-solving & Reasoning (seen as a pair) to achievement standards on the A to E scale. The table below is one example from Year 7:

The Year 7 Mathematics standard elaborations (http://www.qcaa.qld.edu.au/downloads/p_10/ac_math_yr7_se.pdf).

This approach illustrates a mechanism whereby connections between different elements of the curriculum can be made and used, and as such support teachers in interpreting and implementing the *Australian Curriculum: Mathematics*.

Year 7 Mathematics standard elaborations

REVISED DRAFT

		A	B	C	D	E
		The folio of student work has the following characteristics:				
Understanding and skills dimensions	Understanding & Fluency					
	Conceptual understanding	Connection and description of mathematical concepts and relationships in a range of situations, including some that are complex unfamiliar	Connection and description of mathematical concepts and relationships in complex familiar or simple unfamiliar situations	Recognition and identification of mathematical concepts and relationships in simple familiar situations	Some identification of simple mathematical concepts	Statements about obvious mathematical concepts
	Procedural fluency	Recall and use of facts, definitions, technologies and procedures to find solutions in a range of situations including some that are complex unfamiliar	Recall and use of facts, definitions, technologies and procedures to find solutions in complex familiar or simple unfamiliar situations	Recall and use of facts, definitions, technologies and procedures to find solutions in simple familiar situations	Some recall and use of facts, definitions, technologies and simple procedures	Partial recall of facts, definitions or simple procedures
	Mathematical language and symbols	Effective and clear use of appropriate mathematical terminology, diagrams, conventions and symbols	Consistent use of appropriate mathematical terminology, diagrams, conventions and symbols	Satisfactory use of appropriate mathematical terminology, diagrams, conventions and symbols	Use of aspects of mathematical terminology, diagrams and symbols	Use of everyday language



		A	B	C	D	E	
The folio of student work has the following characteristics:							
Understanding and skills dimensions	Problem solving & Reasoning	Problem-solving approaches	Application of relevant problem-solving approaches to investigate a range of situations, including some that are complex unfamiliar	Application of relevant problem-solving approaches to investigate complex familiar or simple unfamiliar situations	Application of problem-solving approaches to investigate simple familiar situations	Some selection and application of problem-solving approaches in simple familiar situations.	Partial selection of problem-solving approaches
	Mathematical modelling	Development of mathematical models and representations in a range of situations, including some that are complex unfamiliar	Development of mathematical models and representations in complex familiar or simple unfamiliar situations	Development of mathematical models and representations in simple familiar situations	Statements about simple mathematical models and representations	Isolated statements about given mathematical models and representations	
	Reasoning and justification	Clear explanation of mathematical thinking and reasoning, including justification of choices made, evaluation of strategies used and conclusions reached	Explanation of mathematical thinking and reasoning, including reasons for choices made, strategies used and conclusions reached	Description of mathematical thinking and reasoning, including discussion of choices made, strategies used and conclusions reached	Statements about choices made, strategies used and conclusions reached	Isolated statements about given strategies or conclusions	

Note: Colour **highlights** have been used in the table to emphasise the qualities that discriminate between the standards.

Scenario 4

Pre-school numeracy and Powerful Mathematical Ideas

Scenario 4. Pre-school numeracy and Powerful Mathematical Ideas

The *Southern Numeracy Initiative* (SNI) pre-school project (reported in Perry, Dockett & Harley, 2007) provides a comprehensive model of using a set of Big Ideas to create a rubric that can be used on a day-to-day basis in teaching and learning and that is linked to a curriculum framework. A practical scaffold was created, the Numeracy Matrix, that enabled seven powerful mathematical ideas (another set of Big Ideas) in pre-school mathematics to be partnered with eight developmental learning outcomes. The powerful mathematical ideas were:

1. Mathematisation*;
2. Connections;
3. Argumentation;
4. Number sense and mental computation;
5. Algebraic reasoning;
6. Spatial and geometric reasoning;
7. Data and probability sense. (DETE, 2001)

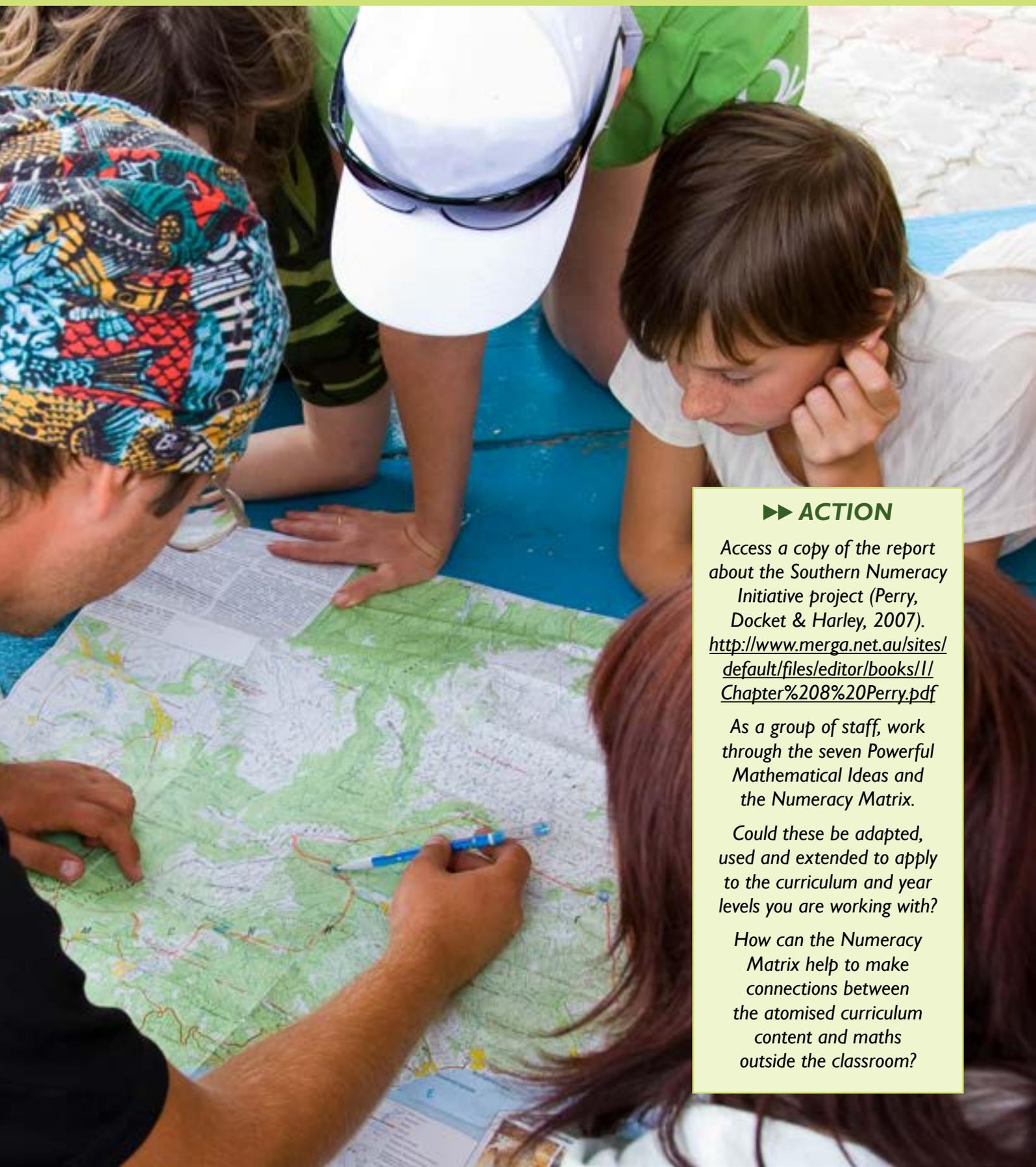
* referred to as Formulate in the PISA framework

These seven Big Ideas were linked with the *South Australian Curriculum, Standards and Accountability (SACSA) Framework* for children's learning and mapped against eight Developmental Learning Outcomes:

1. Children develop trust and confidence.
2. Children develop a positive sense of self and a confident personal and group identity.
3. Children develop a sense of being connected with others and their world.
4. Children are intellectually inquisitive.
5. Children develop a range of thinking skills.
6. Children are effective communicators.
7. Children demonstrate a sense of physical well-being.
8. Children develop a range of physical competencies.

The early childhood educators involved in the project developed a numeracy matrix of pedagogical questions that connected the mathematical ideas and the developmental learning outcomes. Each cell provides examples of pedagogical questions that educators can reflect on, address and use. This Numeracy Matrix enabled focussed pedagogical inquiry questions to be framed and participants reported great gains in attitude, confidence, knowledge and understanding. Below is an example of three of the cells of the numeracy matrix:

Powerful Mathematical Ideas:			
Developmental Learning Outcomes:	<i>Mathematisation</i>	<i>Connections</i>	<i>Algebraic Reasoning</i>
<i>Children develop trust and confidence</i>	<p>How do we encourage children to take risks as they seek to find the mathematics in everyday situations?</p> <p>What opportunities and support do we give children to choose to play with and participate fully in mathematical situations arising from their worlds?</p>	<p>How do we encourage children to play and interact purposefully with the mathematics they experience in their lives?</p> <p>What opportunities and support do we give children to manage and work with different contexts in which a mathematical idea can occur?</p>	<p>How do we encourage children to explore patterns?</p> <p>What opportunities and support do we give children to manage change as they engage with pattern-making activities?</p>



►► ACTION

Access a copy of the report about the Southern Numeracy Initiative project (Perry, Docket & Harley, 2007).

<http://www.merga.net.au/sites/default/files/editor/books/11/Chapter%208%20Perry.pdf>

As a group of staff, work through the seven Powerful Mathematical Ideas and the Numeracy Matrix.

Could these be adapted, used and extended to apply to the curriculum and year levels you are working with?

How can the Numeracy Matrix help to make connections between the atomised curriculum content and maths outside the classroom?

Scenario 5

Using strategies and models

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Scenario 5. Using strategies and models

It is accepted that learning mathematics occurs along a developmental continuum throughout schooling and beyond. Askew (n.d.) (<http://nrich.maths.org/8348>) argues that it is beneficial for students to be introduced early to strategies and models, such as number lines and arrays, which have the capacity to allow their thinking to develop as the curriculum content develops:

A mark of a good model for/tool for thinking with is that it can help learners gain insight into mathematical structure, not simply get correct answers.

The effect is that as students progress they have tried and trusted models of thinking to fall back on. This supports and enables connections to be made among the different elements of a curriculum framework. Some examples of such models include:

- ▶ **Number lines:** from empty lines used to model operations with whole numbers, to representations of fractions on lines marked with positive integers, to acting out addition or subtraction of positive and negative whole numbers, to intersecting number lines that define the Cartesian plane. In the Australian Curriculum: Mathematics their use is referred to across Years 1 to 10. (See Content Descriptions: ACMNA013, ACMNA078, ACMNA102, ACMNA124, ACMNA125, ACMNA152, ACMNA236)
- ▶ **Arrays:** as representations of multiplication and division, from whole numbers to fractions. (ACMNA031, ACMSP225)

- ▶ **Think boards:** from young students representing a worded scenario as a story, as a picture, with manipulatives and as a calculation, extending at secondary level to translating between verbal, graphical, tabular and symbolic representations. Wong (2004) discusses how the model extends to secondary level. Bosse, Adu-Gyamfi and Cheetham (2011) show that teachers tend to doubt students' ability to handle all of these translations, especially in going back from graphical or symbolic forms to verbal ones, thereby undervaluing the literacy component in mathematical understanding. (AC: M Foundation Year Above Satisfactory portfolio work sample 1; Year 1 Above Satisfactory portfolio, work sample 3)
- ▶ **Bar models** or number strips: developed in Singapore and used throughout the years of schooling, initially in comparing number values and exploring equality, then adapting to fractions and extending to ratio and proportion problems which seem less daunting to students because they are using a tried and trusted model. (Liu & Soo, 2014)

As well as physical models, it is important that students develop reliable mental models of processes such as addition and subtraction. This is an instance of another big idea—inverse operations. An informal notion of addition as collecting like items works for whole numbers as adding units to units and tens to tens, extends readily to decimal place value, applies with fractions where there must be like denominators, then like terms in algebra, like surds, like matrices with the same order, and the like parts, real and imaginary, of complex numbers.

▶▶ ACTION

*What strategies and models do you use to support and allow your students' thinking to develop as the curriculum content develops?
Do all team members use the same models across and between year levels?*

Moving forward — coherence and connectedness

Schmidt, Houang and Cogan (2002) describe the advantages that accrue from having a coherent curriculum rather than one that is 'a mile wide and an inch deep'. This need for a sense of coherence or connectedness applies from the stages of writing a national curriculum all the way through to what is happening in each classroom today and tomorrow. The connectedness is such that teachers and students acquire a *mathematical sensibility* both in the sense of connecting mathematics to the real world, and in the sense of exploring the internal coherence and integrity of mathematics as a field of study.

The concept of Big Ideas is powerful because it assists teachers in developing a coherent overview of mathematics. But more importantly it enables students to develop a deeper understanding of mathematics and its interconnectedness, both within the world of mathematics and between the world of mathematics and the real world. Models exist for both.

Over time, the manner in which teachers think about their subject and students relate to it will change.

Effective teachers recognise the connections between different aspects and representations of mathematics. They ask timely and appropriate questions, facilitate and maintain high-level conversations about important mathematics, evaluate and respond to student thinking during instruction, promote understanding, help students make connections, and target teaching to ensure key ideas and strategies are understood. (Siemon et al., 2012, p. 21)

The Vision statement of the Australian Association of Mathematics Teachers captures this notion: *A society of mathematically capable citizens who understand and value mathematics and its contribution to the lives of all Australians.* (AAMT Strategic Plan 2014-2016)

Comment: Big Ideas for mathematics teaching and learning

The concept of Big Ideas ...assists teachers in developing a coherent overview of mathematics ... and enables students to develop a deeper understanding of mathematics and its connectedness.

This edition of the Research Digest reviews a range of research about Big Ideas in mathematics education. The Big Ideas concept argues for an integrated, coherent and connected approach to learning and teaching, which supports students' development of a deep understanding of mathematics, and of the connections between the world of mathematics and the everyday world.

The evidence that mathematical content knowledge and teaching practices should be based on a small number of overarching ideas aligns with the rationale for *the Australian Curriculum: Mathematics*. This approach is also clearly evident in the framework for mathematical literacy in the OECD Program for International Student Assessment (PISA).

The body of research on Big Ideas has clear implications for teaching, and these are explored in this Research Digest. A range of scenarios present practical suggestions for classroom action in varied educational contexts, effectively demonstrating how the Big Ideas concept can provide coherence and focus in day-to-day mathematics teaching in classrooms at all levels.



Useful websites

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Conrad Wolfram

Website: <http://www.conradwolfram.com>

To see and hear Conrad Wolfram's challenging TED talk from 2010:

https://www.ted.com/talks/conrad_wolfram_teaching_kids_real_math_with_computers

NRICH

Website: <http://nrich.maths.org>

The NRICH Project provides learning resources for use by teachers, including rich mathematics-related tasks that show mathematics in meaningful contexts.

Shell Centre

Website: <http://www.mathshell.com/>

For access to the important Shell Centre materials and activities.

Peter Sullivan: Summary of Research on mathematics education

For an overview of recent research on mathematics education practices read Peter Sullivan - Australian Education Review 59 - Teaching Mathematics: Using research-informed strategies.

Website: <http://research.acer.edu.au/cgi/viewcontent.cgi?article=1022&context=aer>

Youcubed

Website: <http://youcubed.stanford.edu/>

Youcubed at Stamford University aims to inspire, educate and empower teachers of mathematics, transforming the latest research on maths learning into accessible and practical forms. The site includes access to papers and online courses run by Jo Boaler.

How to cite this Research Digest

Tout, D. & Spithill, J. (2015). Big Ideas in Mathematics Teaching *The Research Digest*, QCT, 2015 (11)

Retrieved Month, DD, YEAR from <http://www.qct.edu.au>

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